1 The diagram shows a prism *ABCDEFGH* with a horizontal base.

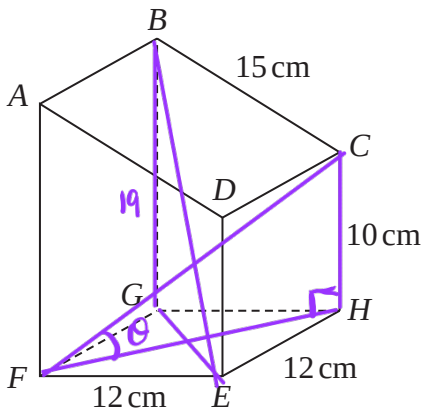


Diagram **NOT** accurately drawn

The base of the prism, *EFGH*, is a square of side 12 cm.
Trapezium *ADEF* is a cross section of the prism where *AF* and *DE* are vertical edges.

DE = *CH* = 10 cm
AD = *BC* = 15 cm

(a) Work out the size of the angle between *CF* and the base *EFGH*.
Give your answer correct to one decimal place.

diagonal FH = $\sqrt{12^2+12^2}$
= $12\sqrt{2}$ cm (1)

By using trigonometry :

$\tan\theta = \frac{10}{12\sqrt{2}}$ (1)

$\theta = 30.5^\circ$ (1)

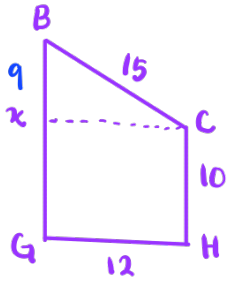
30.5 °

(3)

(b) Work out the length of *BE*.
Give your answer correct to one decimal place.

BX = $15^2 - 12^2$
= 9 cm

BG = 9 + 10 = 19 cm (1)



BE = $\sqrt{BG^2 + GE^2}$
= $\sqrt{19^2 + (12\sqrt{2})^2}$ (1)
= $\sqrt{649}$
= 25.5 cm (1)

BGE forms a right angle triangle.
We can use Pythagoras Theorem to solve for BE.

25.5 cm

(3)

(Total for Question 1 is 6 marks)

2 The diagram shows a cuboid *ABCDEFGH*.

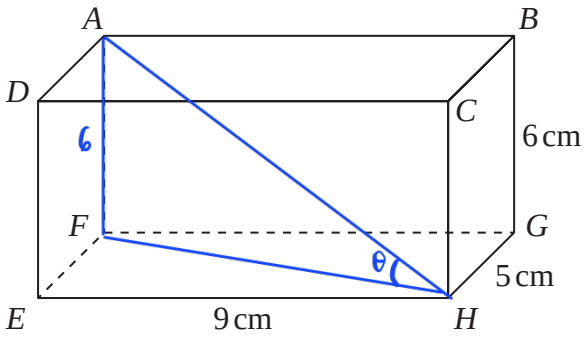


Diagram **NOT** accurately drawn

EH = 9 cm, *HG* = 5 cm and *GB* = 6 cm.

Work out the size of the angle between *AH* and the plane *EFGH*.
Give your answer correct to 3 significant figures.

diagonal FH $= \sqrt{5^2 + 9^2}$

$= \sqrt{106}$ (1)

$\tan\theta = \frac{AF}{FH}$

$\tan\theta = \frac{6}{\sqrt{106}}$ (1)

$\theta = \tan^{-1}\frac{6}{\sqrt{106}}$ (1)

$= 30.2^\circ$ (1)

30.2 °

(Total for Question 2 is 4 marks)

3 The diagram shows a triangular prism *ABCDEF* with a horizontal base *ABEF*.

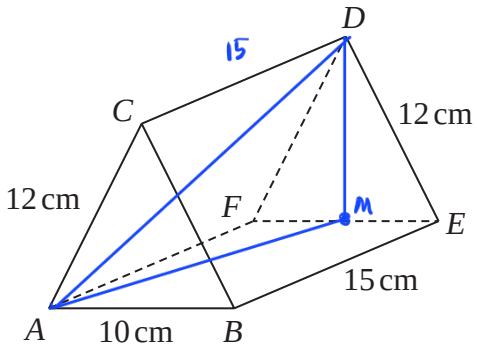


Diagram **NOT** accurately drawn

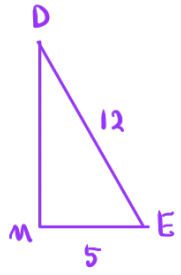
$AC = BC = FD = ED = 12\,\text{cm}$ $AB = 10\,\text{cm}$ $BE = 15\,\text{cm}$

Calculate the size of the angle between *AD* and the base *ABEF*.
Give your answer correct to 3 significant figures.

By using Pythagoras' theorem:

$DM = \sqrt{12^2 - 5^2}$

$= \sqrt{119}$ (1)



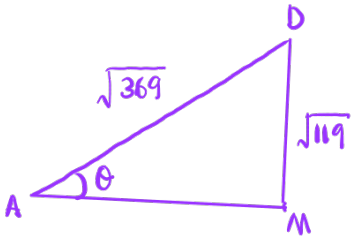
By using diagonal length length formula:

$AD = \sqrt{12^2 + 15^2}$

$= \sqrt{369}$ (1)

Angle between AD and base ABEF:

$\sin\theta = \frac{\sqrt{119}}{\sqrt{369}}$ (1)



$\theta = \sin^{-1}\frac{\sqrt{119}}{\sqrt{369}}$

$= 34.6^\circ$ (1)

34.6 °

(Total for Question 3 is 4 marks)

4 *ABC* is an isosceles triangle in a horizontal plane.
The point *T* is vertically above *B*.

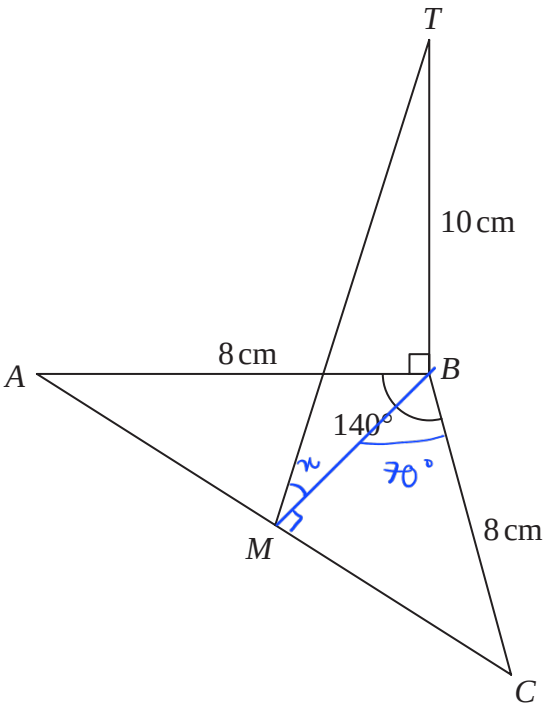
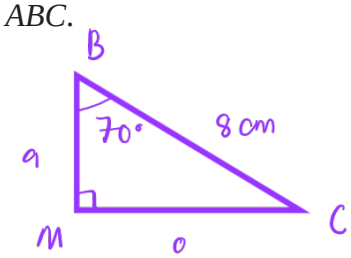


Diagram **NOT** accurately drawn

Angle *ABC* = 140°
AB = *BC* = 8 cm
TB = 10 cm
M is the midpoint of *AC*.

Calculate the size of the angle between *MT* and the horizontal plane *ABC*.
Give your answer correct to one decimal place.

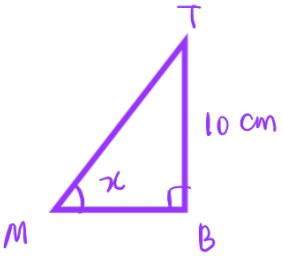


$\cos 70° = \frac{MB}{8}$ (1)

$MB = 8 \cos 70°$

$= 2.73616..$ (1)

$\tan x = \frac{10}{2.73616...}$ (1)



$= 3.6547...$

$x = \tan^{-1}(3.6547...)$

$= 74.7°$ (1dp) (1)

74.7 °

(Total for Question 4 is 4 marks)

5 The diagram shows a solid prism *ABCDEFGH*.

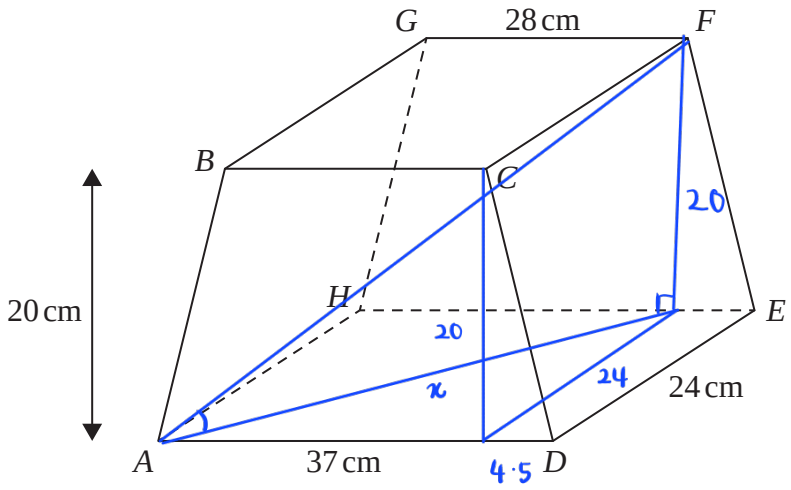


Diagram **NOT** accurately drawn

The trapezium *ABCD*, in which *AD* is parallel to *BC*, is a cross section of the prism.
The base *ADEH* of the prism is a horizontal plane.
ADEH and *BCFG* are rectangles.
The midpoint of *BC* is vertically above the midpoint of *AD* so that *BA* = *CD*.

AD = 37 cm *GF* = 28 cm *DE* = 24 cm

The perpendicular distance between edges *AD* and *BC* is 20 cm.

(b) Calculate the size of the angle between *AF* and the plane *ADEH*.
Give your answer correct to one decimal place.

$x = \sqrt{(37 - 4.5)^2 + (24)^2}$

$= 40.4\ldots$ (1)

$\tan \angle A = \frac{20}{40.4\ldots}$ (1)

$\angle A = \tan^{-1}(0.495\ldots)$

$\angle A = 26.3°$ (1)

26.3 °

(3)

(Total for Question 5 is 3 marks)

6 The diagram shows a cube *ABCDEFGH* with sides of length 6 cm.

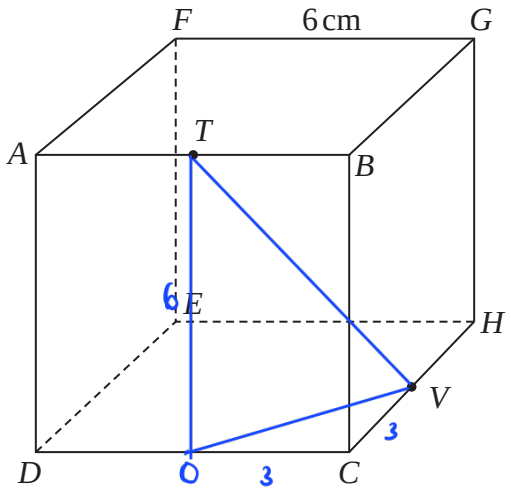
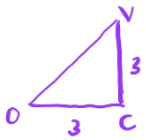


Diagram **NOT** accurately drawn

T is the midpoint of *AB* and *V* is the midpoint of *CH*

Work out the distance from *T* to *V* in a straight line through the cube.
Give your answer in the form $\sqrt{a}$ cm where *a* is an integer.

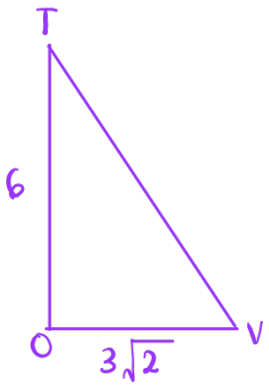
Finding length OV :



$OV = \sqrt{3^2 + 3^2}$ (1)

$= 3\sqrt{2}$ cm (1)

Finding length TV :



$TV = \sqrt{6^2 + (3\sqrt{2})^2}$ (1)

$= \sqrt{54}$ (1)

where $a = 54$

$\sqrt{54}$ ..... cm

(Total for Question 6 is 4 marks)

7 *ABCD* is a horizontal rectangular field.

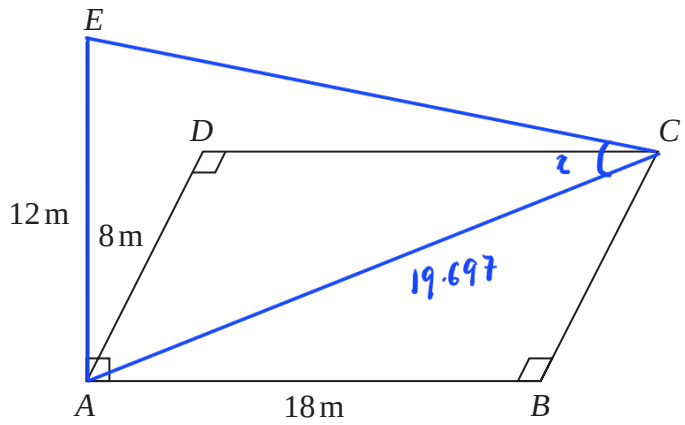


Diagram **NOT** accurately drawn

A vertical pole, *AE*, is placed at the corner *A* of the field.

$AE = 12\,\text{m}$ $AB = 18\,\text{m}$ $AD = 8\,\text{m}$

Calculate the size of the angle between *EC* and the plane *ABCD*
Give your answer correct to one decimal place.

$AC = \sqrt{8^2 + 18^2}$

$= \sqrt{388}$ (1)

$= 19.697\ldots$

$\tan x = \frac{12}{19.697\ldots}$ (1)

$x = \tan^{-1}\frac{12}{19.697\ldots}$

$= 31.4$ (1)

31.4 °

(Total for Question 7 is 3 marks)

8 The diagram shows the prism *ABCDEFGHJK* with horizontal base *AEFG*

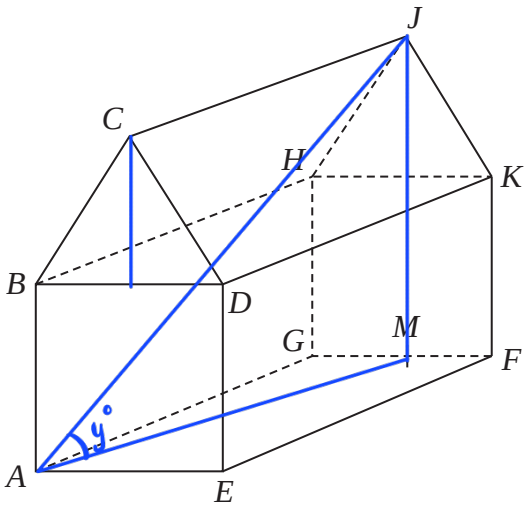


Diagram **NOT** accurately drawn

ABCDE is a cross section of the prism where
ABDE is a square
BCD is an equilateral triangle

$EF = 2 \times AE$
M is the midpoint of *GF* so that *JM* is vertical.

Angle $MAJ = y°$

Given that $\tan y° = T$

find the value of *T*, giving your answer in the form $\frac{\sqrt{p} + \sqrt{q}}{17}$ where *p* and *q* are integers.

let GM = x

GF = $2x$

EF = $4x$

AM = $\sqrt{x^2 + (4x)^2}$

= $\sqrt{17x^2}$ (1)

= $\sqrt{17}\,x$

Height of triangle : $\sqrt{(2x)^2 - x^2}$

= $\sqrt{3}\,x$ (1)

JM = $2x + \sqrt{3}\,x$

$\tan y^\circ = \dfrac{2x + \sqrt{3}\,x}{\sqrt{17}\,x}$ (1)

$\tan y^\circ = \dfrac{2+\sqrt{3}}{\sqrt{17}} = T$

$\dfrac{2+\sqrt{3}}{\sqrt{17}} \times \dfrac{\sqrt{17}}{\sqrt{17}}$ (1)

$\dfrac{2\sqrt{17} + \sqrt{51}}{17}$

$= \dfrac{\sqrt{4\times17} + \sqrt{51}}{17}$

$= \dfrac{\sqrt{68} + \sqrt{51}}{17}$ (1)

$T = \dfrac{\sqrt{68} + \sqrt{51}}{17}$

(Total for Question 8 is 5 marks)

9 The diagram shows a triangular prism, *ABCDEF*, with a rectangular base *ABCD*

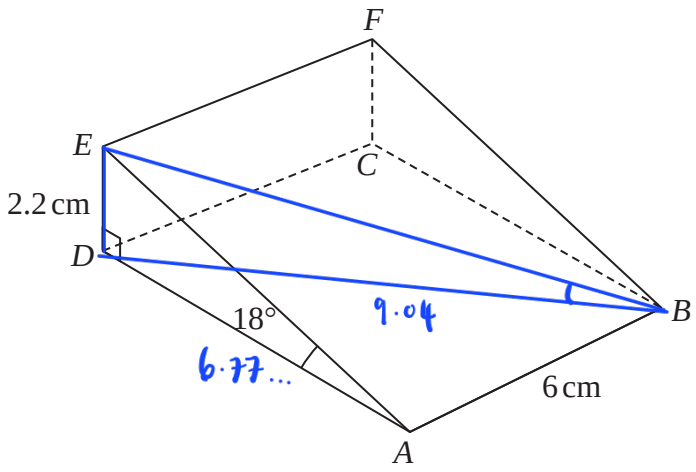


Diagram **NOT** accurately drawn

AB = 6 cm *DE* = 2.2 cm angle *DAE* = 18° angle *ADE* = 90°

Work out the angle that *BE* makes with the plane *ABCD*
Give your answer correct to one decimal place.

$\tan 18^\circ = \frac{2.2}{AD}$

$AD = \frac{2.2}{\tan 18^\circ} = 6.77\ldots$ (1)

$BD = \sqrt{6.77\ldots^2 + 6^2}$ (1)

$= 9.04\ldots$

$\tan DBE = \frac{2.2}{9.04\ldots}$ (1)

$DBE = \tan^{-1} \frac{2.2}{9.04\ldots}$

$= 13.7$ (1)

13.7 °

(Total for Question 9 is 4 marks)

10 The diagram shows a solid prism *ABCDEFGHIJ*

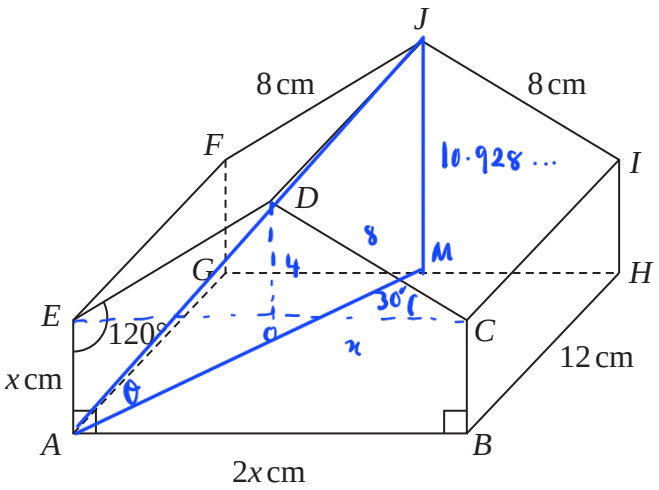


Diagram **NOT** accurately drawn

The prism is such that each cross section is a pentagon where

$AE = BC = x$ cm $\quad AB = 2x$ cm $\quad ED = CD = 8$ cm

angle EAB = angle $CBA = 90°$ angle AED = angle $BCD = 120°$

Given that $AG = BH = EF = DJ = CI = 12$ cm

calculate the angle that AJ makes with the base $ABHG$ of the prism.
Give your answer correct to 3 significant figures.

$DO = 8 \sin 30^\circ$

$= 4$ (1)

$x = 8 \cos 30^\circ = 6.928\ldots$ (1)

$2x = 2 \times 6.928\ldots = 13.85\ldots$

$AM = \sqrt{12^2 + 6.928\ldots^2}$ (1)

$= 13.856\ldots$

$JM = 4 + 6.928\ldots = 10.928\ldots$

$\tan\theta = \dfrac{10.928\ldots}{13.856\ldots}$ (1)

$\theta = \tan^{-1}\dfrac{10.928\ldots}{13.856\ldots}$

$\theta = 38.3$ (1)

38.3 °

(Total for Question 10 is 5 marks)

11 Here is a cuboid *PQRSTUVW*

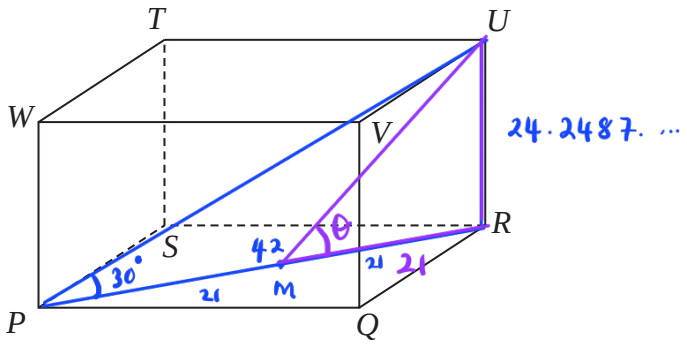


Diagram **NOT** accurately drawn

PR = 42 cm

The size of the angle between *PU* and the plane *PQRS* is 30°

M is the midpoint of *PR*

(b) Work out the size of angle *UMR*
Give your answer correct to 3 significant figures.

$\tan 30^\circ = \frac{UR}{42}$

$UR = 42 \tan 30^\circ$ (1)

$= 24.2487....$

$\tan \theta = \frac{24.2487...}{21}$ (1)

$\tan \theta = 1.1547...$

$\theta = \tan^{-1} 1.1547....$

$= 49.1$ (3 s.f.) (1)

49.1 °

(3)

(Total for Question 11 is 3 marks)

12 The diagram shows a triangle *ABC* and a flagpole *BF*

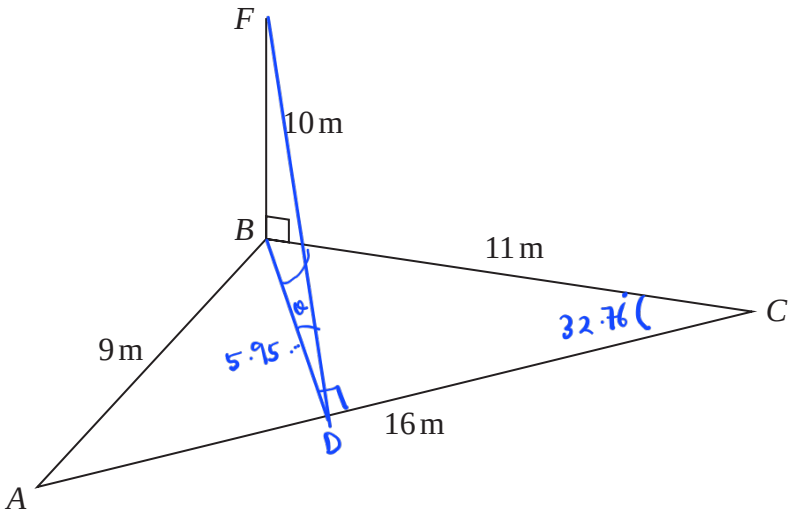


Diagram **NOT** accurately drawn

A, *B* and *C* are points on horizontal ground.

BF is vertical.

$AB = 9\,\text{m}$ $BC = 11\,\text{m}$ $AC = 16\,\text{m}$ $BF = 10\,\text{m}$

D is the point on *AC* such that angle *BDC* = 90°

Work out the size of the angle of elevation of the point *F* from the point *D*
Give your answer correct to one decimal place.

$9^2 = 11^2 + 16^2 - 2(11)(16)\cos BCA$ (1)

$\dfrac{9^2 - 11^2 - 16^2}{-2(11)(16)} = \cos BCA$

$BCA = 32.763\ldots$ (1)

$\dfrac{BD}{\sin 32.763..} = \dfrac{11}{\sin 90}$

$BD = 11 \sin 32.763$

$= 5.95\ldots$ (1)

$\tan FDB = \dfrac{10}{5.95\ldots}$ (1)

$FDB = \tan^{-1}\dfrac{10}{5.95\ldots} = 59.2$ (1)

59.2 °

(Total for Question 12 is 5 marks)